lect 14 Multi-view geometry problem

Given a point in one image, and its 3D location, what's its location in another image?

Epipolar geometry

Use only two cameras

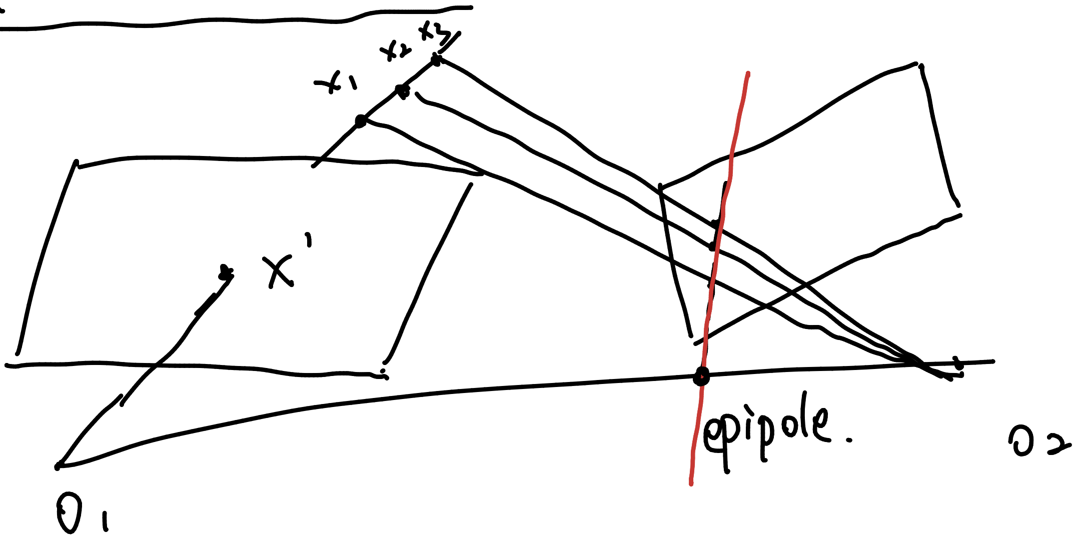
Baseline. a line connecting two cameras' centers

Epipolar Plane. one plane containing baseline
(1-D Family. because only 1-D DoF)

Epipoles:
Intersection of baselines with image plane.
<=> projection of one camera center onto another's plane.
<=> vanish point of the motion direction
from one origion to another

Epipolar Line:
line intersected by epipolar plane & image plane

Epipolar Constraint



if we have x' in left image. in real world its location can be X_1, X_2, X_3 ..... but, its corresponding point must be on a line (red line), which is the epipolar line. And, the line will always pass epipole.

Mathematical Expression (For Calibration Case)

For left camera, construct the world coordinate system.
the projection matrix $P = K[I|0]$

For right camera, there will be a $P' = K'[R|t]$

X : 3-D location, x': location on left image

$\boxed{X_{normal}} = K^{-1}x' = K^{-1}K[I|0]X = [I|0]X$ ← normalized point, with focus length = 1,

$X'_{normal} = K'^{-1}x'' = [R|t]X$ ← no pixel size effect

* Converging Camera:

When two cameras are looking at the same thing

* Motion parallel to image plane

* Motion perpendicular to film

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all epipolar lines will intersect at epipole.

Now, we have a 1 3D-point $\underline{X} = (x, 1)^T$

2 translation t of camera centers

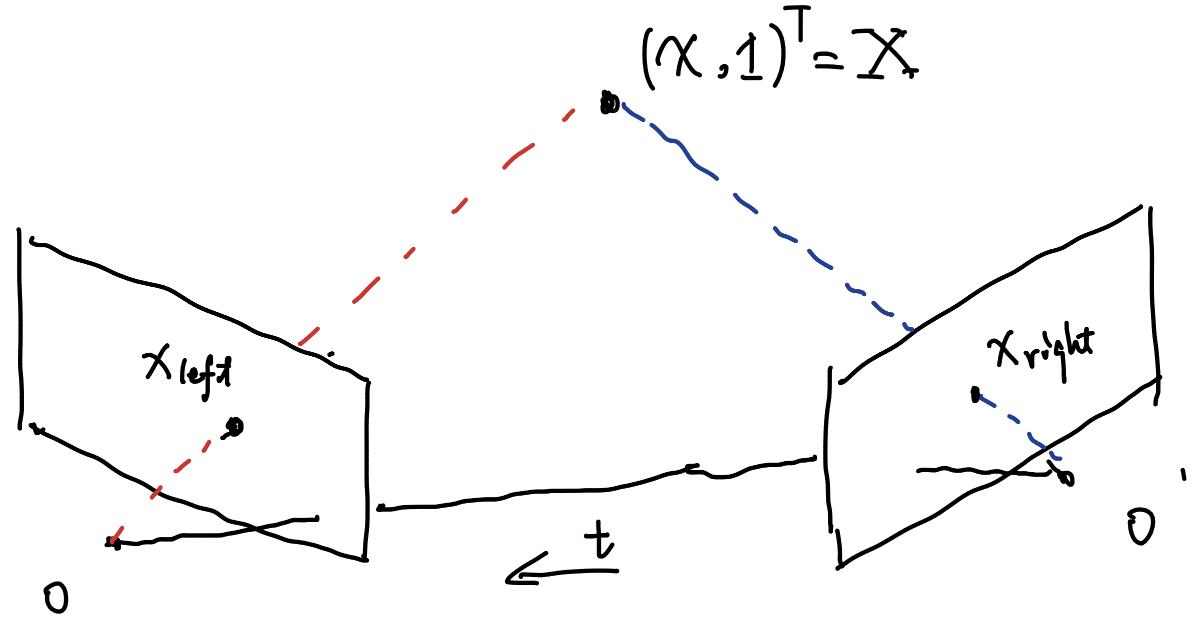
3 Rotation matrix R

for left camera. the image point for $\underline{X}$ is

$x_{left} = [I \mid 0] \begin{pmatrix} x \\ 1 \end{pmatrix}$ Recall, we have use normalized point

Homogenious Coordinate

$x_{right} = [R \mid t] \begin{pmatrix} x \\ 1 \end{pmatrix}$



the $x_{left} = [I \,\vdots\, 0]\begin{pmatrix} x \\ 1 \end{pmatrix}$ is also the direction of line Ox_{left}, because it passes the O-point

$x_{right} = [R \,\vdots\, t]\begin{pmatrix} x \\ 1 \end{pmatrix}$ but! it's in another coordinate system

red vector can be written as $R \cdot X_{left}$ (in right camera's system)

because it should be $RX + t$, but $x_{left} = x$

So RX_{left}, t, $x_{right} - O'$ are co-planer

So we have

$$x_{right}^T \cdot (t \times (R \cdot X_{left})) = 0$$

$$= x_{right}^T \cdot \underbrace{[t_\times]}_{E} R \cdot X_{left}$$

Essential Matrix

$$E = [t_\times] R$$

definition in next page

$$= x_{right}^T E \, X_{left} = 0$$

formula for cross-product

$$a = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad b = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$a \times b = \begin{bmatrix} \hat{i} & \hat{j} & k \\ u & v & w \\ x & y & z \end{bmatrix}$$

$$= \begin{bmatrix} vz - wy \\ uz - wx \\ uy - vx \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 0 & -w & v \\ -w & 0 & u \\ -v & u & 0 \end{bmatrix}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{b}$$

notation

$$= \begin{bmatrix} a_\times \end{bmatrix} b$$

For essential matrix E

$E x_{left}$ is the line epipolar line associate with x_{left}

$E^T x_{right}$ is the epipolar line associate x_{right}

$Ee = E^T e' = 0 \Rightarrow e, e'$ is in the null space of E

epipole

E has at least 1-D Null-space.

$\because E = t \times R$, so $E \cdot \underbrace{(\alpha t + \beta R)}_{\text{one plane}} = 0$, so

$\therefore E$ is a rank-2 singular matrix.

E has 5-DoF if we treat it as projection matrix, or 6 DoF

For Unknown Calibration Case

$$\hat{x}_{right}^T E \hat{x}_{left} = 0$$

$$(K'^{-1} x_{right})^T \qquad (K^{-1} x_{left})$$

We can rewrite it as

$$x_{right}^T \underbrace{K'^{-T} E K^{-1}}_{F} x_{right}$$

F: Fundamental Matrix

F shares the properities with E. but have 7-DoF.

because 9-elements in all,
rank-2,
scaling doesn't matter $\} \rightarrow$ 7 DoF
or, 8-DoF only with constrain of rank-2.

Eight-point Algorithm, minimize the algebraic distance

$x_{left} = (u, v, 1)$

$x_{right} = (u', v', 1)$

① $$\min \sum_{points} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}^T \begin{pmatrix} f_{11} & \cdots & \\ \vdots & \ddots & \\ \vdots & & f_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$ Linear method

s.t. $\left\| \begin{matrix} f_{11} & & \\ & \ddots & \\ & & f_{33} \end{matrix} \right\|_2 = 1$ There are 8- unknown of $\{f_{ij}\}$

② $$\min \sum_{points} \left(d^2(x_{left}, x_{right}^T \cdot F) + d^2(x_{right}, F \cdot x_{left}) \right)$$

minimize geometric distance.

Problem for Eight-point algorithm

$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}^T (F) \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$ will have very large numerical range.

Solution

1. Scale the image and make the mean square distance between pixel and origion to be 2. Because originally, $(u \cdot u')$ may range from 1 ~ Million

2. Apply 8-point algorithm

3. Use SVD and throw away third singular value to enhance Rank-2 property

But:

Estimate "F" is called "weak calibration". because F is a composition of k, k', E. But if we know k, k', we can recover E out